

Nucleon Spin

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Abstract

We review the theory of polarized deep inelastic scattering in light of the most recent experimental data. We discuss the nucleon spin decomposition and the Bjorken sum rule. The latter is used for extraction of $\alpha_s(M_Z^2) = 0.116^{+0.004}_{-0.006}$ and as a test case for a new method of analyzing divergent perturbation series in QCD.

1. Analysis of Polarized Structure Functions

1.1. Formalism

The basis for our discussion will be the two spin-dependent structure functions G_1 and G_2 :

$$\frac{d^2\sigma^{\uparrow\downarrow}}{dQ^2 d\nu} - \frac{d^2\sigma^{\uparrow\uparrow}}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^2 E^2} \left[M_N(E + E' \cos \theta) G_1(\nu, Q^2) - Q^2 G_2(\nu, Q^2) \right] \quad (1)$$

In the parton model, these structure functions scale as follows in the Bjorken limit $x = Q^2/2M_N\nu$ fixed,

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$Q^2 \rightarrow \infty$:

$$\begin{aligned} M_N^2 \nu G_1(\nu, Q^2) &\equiv g_1(x, Q^2) \rightarrow g_1(x) \\ M_N \nu^2 G_2(\nu, Q^2) &\equiv g_2(x, Q^2) \rightarrow g_2(x) \end{aligned} \quad (2)$$

We will discuss the scaling structure function g_2 later on, focussing for now on g_1 , which is related to the polarized quark distributions by

$$\begin{aligned} g_1^p(x) &= \frac{1}{2} \sum_q e_q^2 [q_\uparrow(x) - q_\downarrow(x) + \bar{q}_\uparrow(x) - \bar{q}_\downarrow(x)] \\ &= \frac{1}{2} \sum_q \Delta q(x) \end{aligned} \quad (3)$$

for comparison, the unpolarized structure function F_2 is given by

$$F_2(x) = \sum_q e_q^2 x [q_\uparrow(x) + q_\downarrow(x) + \bar{q}_\uparrow(x) - \bar{q}_\downarrow(x)] \quad (4)$$

so that the polarization asymmetry A_1 may be written as

$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \quad (5)$$

in the Bjorken limit, where $\sigma_{1/2}$ and $\sigma_{3/2}$ are the virtual photon absorption cross sections. We will discuss later the Q^2 dependences of the above formulae, as well as the transverse polarization asymmetry.

Much of the interest in the polarized structure function g_1 is due to its relation to axial current matrix elements:

$$\begin{aligned} \langle p | A_\mu^q | p \rangle &= \langle p | \bar{q} \gamma_\mu \gamma_5 q | p \rangle = \langle p | \bar{q}_R \gamma_\mu q_R - \bar{q}_L \gamma_\mu q_L | p \rangle = \\ &= \Delta q \cdot S_\mu(p) \end{aligned} \quad (6)$$

where $q_{L,R} \equiv 1/2(1 \mp \gamma_5)q$, S_μ is the nucleon spin four-vector, and

$$\Delta q \equiv \int_0^1 dx [q_\uparrow(x) - q_\downarrow(x) + \bar{q}_\uparrow(x) - \bar{q}_\downarrow(x)] \quad (7)$$

Of particular interest is the matrix element of the singlet axial current

$$A_\mu^0 = \sum_{q=u,d,s} \bar{q} \gamma_\mu \gamma_5 q : \quad \langle p | A_\mu^0 | p \rangle = \sum_{q=u,d,s} \Delta q \cdot S_\mu(p) \quad (8)$$

which is related in the parton model to the sum of the light quark contributions to the proton spin. Prior to the series of measurements of polarized deep inelastic lepton nuclear scattering, information was available from charged current weak interactions on some axial current matrix elements. For example, neutron beta decay and strong isospin symmetry tell us that [1]

$$\Delta u - \Delta d = F + D = 1.2573 \pm 0.0028 \quad (9)$$

and hyperon beta decays and flavour $SU(3)$ symmetry tell us that [2]

$$\frac{\Delta u + \Delta u - 2\Delta s}{\sqrt{3}} \equiv \frac{a_8}{\sqrt{3}} = \frac{3F - D}{\sqrt{3}} = 0.34 \pm 0.02 \quad (10)$$

(for a recent discussion of the applicability of $SU(3)$ symmetry see Ref. [3] and references therein). Equations (9) and (10) give us two equations for the three unknowns Δu , Δd and Δs . In principle, a third piece of information was available [4, 5] in 1987 from neutral current weak interactions. Measurements of νp and $\bar{\nu} p$ elastic scattering [6] indicated that

$$\Delta s = -0.15 \pm 0.09 \quad (11)$$

but this information was not generally appreciated before the advent of the EMC data discussed below. At present there is a new neutrino experiment under way at Los-Alamos [7] which is expected to significantly improve the precision of (11) (see Ref. [8] for a recent in-depth analysis).

In the naïve parton model, the integrals of the g_1 structure functions for the proton and neutron

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2) \quad (12)$$

$$\Gamma_1^n(Q^2) \equiv \int_0^1 dx g_1^n(x, Q^2)$$

are related to combinations of the Δq .

$$\Gamma_1^p = \frac{1}{2} \left(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right) \quad (13)$$

$$\Gamma_1^n = \frac{1}{2} \left(\frac{1}{9} \Delta u + \frac{4}{9} \Delta d + \frac{1}{9} \Delta s \right)$$

The difference between the proton and neutron integrals yields the celebrated Bjorken sum rule [9]

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{1}{6} (\Delta u - \Delta d) \times (1 - \alpha_s(Q^2)/\pi) + \dots \quad (14)$$

It is not possible to derive individual sum rules for $\Gamma_1^{p,n}$ without supplementary assumptions. The assumption made by Ellis and Jaffe in 1973 [10] was that $\Delta s = 0$, on the grounds that very possibly there were a negligible number of strange quarks in the nucleon wave function, and if there were, surely they would not be polarized. With this assumption, it was estimated that

$$\begin{aligned} \int_0^1 dx g_1^p(x, Q^2) &= \frac{1}{18} (4\Delta u + \Delta d) (1 - \alpha_s/\pi + \dots) = \\ &= 0.17 \pm 0.01 \end{aligned} \quad (15)$$

It should be clear that this was never a rigorous prediction, and was only intended as a qualitative indication to experimentalists of what they might find when they started to do polarized electron proton scattering experiments.

Perturbative QCD corrections to the above relations have been calculated [11]-[13]:

$$\int_0^1 [g_1^p(x, Q^2) - g_1^n(x, Q^2)] = \frac{1}{6} |g_A| f(x) : \quad (16)$$

$$f(x) = 1 - x - 3.58x^2 - 20.22x^3 + \dots$$

and

$$\begin{aligned} \int_0^1 g_1^{p(n)}(x, Q^2) &= \\ &= \left(\pm \frac{1}{12} |g_A| + \frac{1}{36} a_8 \right) f(x) + \frac{1}{9} \Delta \Sigma(Q^2) h(x) : \quad (17) \\ h(x) &= 1 - x - 1.096x^2 - \dots \end{aligned}$$

where $x = \alpha_s(Q^2)/\pi$, and the dots represent uncalculated higher orders of perturbation theory, to which must be added higher-twist corrections which we will discuss later. The coefficients in (16),(17) are for $N_f=3$, as relevant for the Q^2 range of current experiments.

With these corrections, the Bjorken sum rule is a fundamental prediction of QCD which can be used, for example, to estimate a value for $\alpha_s(Q^2)$. On the other hand, the individual proton and neutron integrals can be used to extract a value of Δs .

1.2. The Helen of spin

Early data on polarized electron-proton scattering from SLAC-Yale experiments [14, 15, 16] were compatible with the prediction of equation (15) within large errors. Over a 1000 theoretical and experimental papers were launched by the 1987 EMC result [17]

$$\begin{aligned} \int_0^1 g_1^p(x, Q^2) &= 0.126 \pm 0.010 \text{ (syst.)} \pm 0.015 \text{ (stat.)} , \\ \text{at } \langle Q^2 \rangle &= 10.7 \text{ GeV}^2 \end{aligned} \quad (18)$$

which was in *prima facie* disagreement with the dynamical assumption that $\Delta s = 0$. It is worth pointing out that the small- x behaviour of $g_1^p(x)$ was crucial to this conclusion. The earlier SLAC-Yale data had large extrapolation errors, and the EMC data indicated behaviour different from that in simple dynamical models. They were, however, consistent [4] with the naïve Regge expectation [18]

$$g_1^p(x) \simeq \sum_i c_i x^{-\alpha_i(0)} \quad (19)$$

were the $\alpha_i(0)$ are the intercepts of axial vector Regge trajectories which are expected to lie between 0 and -0.5. A fit to the EMC data gave [4]

$$g_1^p \sim x^{-\delta} : \quad \delta = -0.07_{-0.32}^{+0.42} \text{ for } x < 0.2 \quad (20)$$

for $x < 0.2$.

Using equations (9),(10),(18) and the leading order perturbative QCD corrections in equation (17) it was estimated [19] that

$$\begin{aligned} \Delta u &= 0.78 \pm 0.06 \\ \Delta d &= -0.47 \pm 0.06 \\ \Delta s &= -0.19 \pm 0.06 \end{aligned} \quad (21)$$

Strikingly, these determinations corresponded to a total contribution of quarks to the proton spin

$$\Delta \Sigma = \Delta u + \Delta d + \Delta s = 0.12 \pm 0.17 \quad (22)$$

which was compatible with 0. This has sometimes been called the “proton spin crisis”, but we think this is an over-reaction. The result equation (22) was certainly a surprise for naïve models of non-perturbative QCD, but it was not in conflict with perturbative QCD. Moreover, shortly after the first data became available it was shown [19] that $\Delta \Sigma = 0$ occurs naturally in the Skyrme model, which is believed to reproduce the essential features of QCD in the large- N_c limit. Alternatively, it was suggested that the $U(1)$ axial anomaly and polarized glue might provide an alternative interpretation [20]-[22], or a significant suppression of the QCD topological susceptibility [23]-[25] might play a key rôle, which would modify the naïve quark model predictions.

In 1972 Richard Feynman wrote “... *its [the Bjorken sum rule’s] verification, or failure, would have a most decisive effect on the direction of future high-energy physics*”. On the other hand, we think that the verification, or failure, of equation (22) has only an indecisive effect, though a very interesting one.

1.3. Evaluation of integrals

Before discussing the interpretation of more recent data on polarized structure functions, we first review a few points that arise in the evaluation of the integrals $\Gamma_1^{p,n}$. It should not be forgotten that the QCD versions of the sum rules are formulated at fixed Q^2 . A generic deep-inelastic sum rule in QCD reads

$$\Gamma(Q^2) = \Gamma_\infty \left[1 + \sum_{n \geq 1} c_n \left(\frac{\alpha_s(Q^2)}{\pi} \right)^n \right] + \sum_{m \geq 1} \frac{d_m}{(Q^2)^m} \quad (23)$$

where Γ_∞ is the asymptotic value of the sum rule for $Q^2 \rightarrow \infty$, the c_n are the coefficients of the perturbative corrections, and the d_m are coefficients of the so-called mass and higher-twist corrections. On the other hand, the data are normally obtained at values of Q^2 that increase on the average with x_{Bj} as seen in Fig. 1.

It is therefore necessary for each individual experiment to interpolate and extrapolate to some fixed mean value of Q^2 , as indicated by the dashed horizontal line in Fig. 1. The quantity measured directly is the polarization asymmetry (5), which seems experimentally to

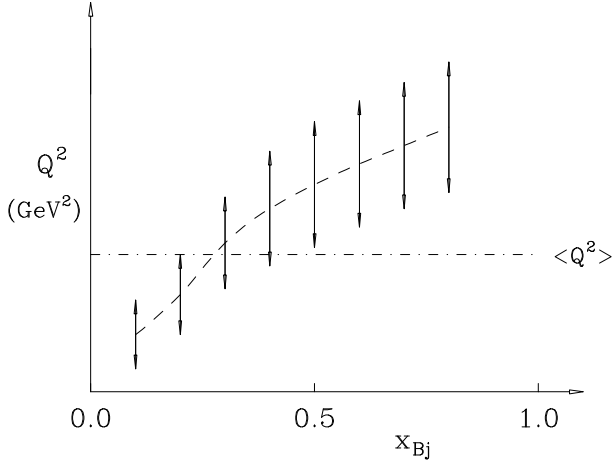


Figure 1. In any given polarized lepton-nucleon scattering experiment, the range of Q^2 probed is different in different bins of the Bjorken variable x_{Bj} .

have only small dependence on Q^2 . Therefore experiments often assume that A_1 is a function of x only, and then estimate

$$g_1(x, Q^2) = \frac{A_1(x, Q^2)F_2(x, Q^2)}{2x[1 + R(x, Q^2)]} \simeq \frac{A_1(x)F_2(x, Q^2)}{2x[1 + R(x, Q^2)]} \quad (24)$$

where $F_2(x, Q^2)$ and $R(x, Q^2)$ (the ratio of longitudinal to transverse virtual photon cross-sections) are taken from parametrizations of unpolarized scattering data. Note that these induce a Q^2 dependence in g_1 even if A_1 is independent of Q^2 .

The possible reliability of the assumption that A_1 is independent of Q^2 can be explored using leading-order perturbative QCD models for $g_1(x, Q^2)$. Several such studies have been made [26]–[31], and they indicate a small Q^2 dependence in $A_1(x, Q^2)$. This has in turn a small effect on the extracted values of $g_1(x, Q^2)$, which is much smaller than the statistical errors of the EMC and SMC experiments and is not very significant for the E142 and E143 experiments, and is in any case considerably smaller than the systematic errors, so that it does not yet contribute an important error to the evaluations of the $\Gamma_1^{p,n}$. However, it could become an important effect in the future, and both theorists and experimentalists should keep their eyes open.

The old-fashioned assumption of Regge behaviour at low x also needs to be checked carefully. The leading-order perturbative QCD evolution equations for the non-singlet part of the helicity distributions, $\Delta q_{NS}(x, Q^2)$, lead us to expect [32, 33] singular

behaviour as $x \rightarrow 0$, so that

$$\Delta q_{NS}(x, Q^2) \simeq C_{NS} \exp(A_{NS}\sigma + B_{NS}\frac{\sigma}{\rho} - \ln \rho - \frac{1}{2} \ln \sigma) \quad (25)$$

where A_{NS} , B_{NS} and C_{NS} are some constants and

$$\sigma \equiv \sqrt{\ln \frac{x_0}{x} \ln \frac{t}{t_0}}, \quad \rho \equiv \sqrt{\frac{\ln \frac{x_0}{x}}{\ln \frac{t}{t_0}}}, \quad t \equiv \left(\frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_0^2}{\Lambda^2}} \right) \quad (26)$$

and we might expect by analogy with the unpolarized structure functions that $x_0 \sim 0.1$, $Q_0^2 \sim 1$ GeV and the leading-order QCD scale parameter $\Lambda \sim 0.25$ GeV, with

$$A_{NS} = \frac{4\sqrt{2}}{\sqrt{33 - 2N_f}}, \quad B_{NS} = \frac{4}{33 - 2N_f}. \quad (27)$$

where $N_f = 3$ in the Q^2 range of current experimental interest (see also Ref. [34]). In principle Eq. (25) can be applied directly to the low- x behavior of the integrand of the Bjorken sum rule: $g_1^p(x, Q^2) - g_1^n(x, Q^2) = \frac{1}{6}[\Delta u(x, Q^2) - \Delta d(x, Q^2)]$, as well as to the other nonsinglet combination $\Delta u(x, Q^2) + \Delta d(x, Q^2) - 2\Delta s(x, Q^2)$ that also contributes to $g_1^{p,n}(x, Q^2)$. Note, however, that effects similar those in the BFKL pomeron may also be important at very low x [35]. The flavour-singlet combination of structure functions has a more complicated low- x behaviour, which could be important for the extraction of the Δq . More singular low- x behaviours have been proposed in the literature motivated by non-perturbative QCD considerations [36, 37]. It is not clear whether the behaviour in equation (25) is relevant to the data presently available: one SMC data point may be in its region of applicability, and could in principle be used to normalize the perturbative QCD formula, serving as a basis for extrapolating the integrals to $x = 0$. In practice, it does not seem at present that this would have a significant effect on the evaluation of the Bjorken sum rule.

The analysis of the polarized structure function data has often assumed that the transverse polarization asymmetry

$$A_{\perp} = \frac{d\sigma^{\uparrow\rightarrow} - d\sigma^{\downarrow\rightarrow}}{d\sigma^{\uparrow\rightarrow} + d\sigma^{\downarrow\rightarrow}}. \quad (28)$$

is negligible. This is related to the spin-flip photon absorption asymmetry

$$A_2 = \frac{2\sigma_{TL}}{\sigma_{1/2} + \sigma_{3/2}} \quad (29)$$

and the longitudinal A_1 asymmetry (5) through the relation:

$$A_{\perp} = d \left(A_2 - \gamma \left(1 - \frac{\gamma}{2} \right) A_1 \right). \quad (30)$$

were $\sigma_{1/2}$ and $\sigma_{3/2}$ are the virtual photon-nucleon absorption cross sections for total helicity 1/2 and

3/2, respectively, σ_{TL} arises from the helicity spin-flip amplitude in forward photon-nucleon Compton scattering, $\gamma = 2Mx/\sqrt{Q^2}$, and $y = \nu/E_{lepton}$, where ν is the energy transfer in the laboratory frame. The coefficient d is related to the virtual photon depolarization factor D by

$$d = D \frac{\sqrt{1-y}}{1-y/2} \quad (31)$$

The asymmetries A_1 and A_2 are subject to the following positivity conditions [38]

$$|A_1| < 1, \quad |A_2| \leq \sqrt{R}. \quad (32)$$

and are related to the structure functions $g_{1,2}$ by

$$A_1 = \frac{1}{F_1}(g_1 - \gamma^2 g_2), \quad A_2 = \frac{\gamma}{F_1}(g_1 + g_2), \quad (33)$$

where $F_1 = F_2(1+\gamma^2)/2x(1+R)$ is the spin-independent structure function. Recently, data on g_2 have become available for the first time [39, 40]. They indicate that it is considerably smaller than the positivity bound in equation (32), and is very close to the leading twist formula of ref. [41]:

$$g_2^{ww}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 g_1(t, Q^2) \frac{dt}{t}, \quad (34)$$

The data on g_2 are also compatible with the Burkhardt-Cottingham sum rule [42]

$$\int_0^1 g_2(x, Q^2) dx = 0, \quad (35)$$

which has been verified to leading order in perturbative QCD. [43, 44]. However, the experimental errors are still considerable, in particular because the low- x behaviour of g_2 is less well understood than that of g_1 . However, the data already tell us that the uncertainty in g_2 is not significant for the evaluations of the $\Gamma_1^{p,n}$.

1.4. Higher orders in QCD perturbation theory:

The perturbation series in QCD is expected to be asymptotic with rapidly growing coefficients:

$$S(x) = \sum_{n=0}^{\infty} c_n x^n, \quad x \equiv \frac{\alpha_s}{\pi}, \quad c_n \simeq n! K^n n^\gamma \quad (36)$$

for some coefficients K, γ [45, 46]. This type of behaviour is associated with the presence of the renormalon singularities, as we shall discuss shortly. Such series are often evaluated approximately by calculating up to the ‘‘optimal’’ order, implicitly defined by

$$|c_{n_{opt}} x^{n_{opt}}| < |c_{n_{opt}+1} x^{n_{opt}+1}| \quad (37)$$

and assuming an error of the same order of magnitude as $c_{n_{opt}} x^{n_{opt}}$. The question arises whether one can approach or even surpass this accuracy without calculating all the terms up to order n_{opt} . This possibility has been studied using the effective charge (ECH) approach [47, 48] and using commensurate scale relations [49, 50]. In this section, we discuss the use of Padé approximants (PA’s) for this purpose [51, 52].

Padé approximants [53, 54] are rational functions chosen to equal the perturbative series to the order calculated:

$$[N/M] = \frac{a_0 + a_1 x + \dots + a_N x^N}{1 + b_1 x + \dots + b_M x^M} : \quad (38)$$

$$[N/M] = S + \mathcal{O}(x^{N+M+1})$$

Under certain circumstances, an expansion of the PA in equation (38) provides a good estimate, c_{N+M+1}^{est} , the Pade Approximant Prediction (PAP), for the next coefficient c_{N+M+1} in the perturbative series [55]. For example, we have demonstrated that if

$$\epsilon_n \equiv \frac{c_n c_{n+2}}{c_{n+1}^2} - 1 \simeq \frac{1}{n}, \quad (39)$$

as is the case for any series dominated by a finite number of renormalon singularities, then $\delta_{[N/M]}$ defined by

$$\delta_{[N/M]} \equiv \frac{c_{N+M+1}^{est} - c_{N+M+1}}{c_{N+M+1}} \quad (40)$$

has the following asymptotic behaviour

$$\delta_{[N/M]} \simeq -\frac{M!}{L^M}, \quad \text{where } L = N + M + aM \quad (41)$$

and where a is a number of order 1 that depends on the series under consideration. This prediction agrees very well with the known errors in the PAP’s for the QCD vacuum polarization D function calculated in the large N_f approximation [56], as seen in Fig. 2a.

Large- N_f calculations of the perturbative corrections to the Bjorken sum rule [57] indicate the presence of only a finite number of renormalon singularities, so that the PAP’s should be accurate. Using the known terms in equation (16), the [1/2] and [2/1] PAP’s yield the following estimates for the fourth-order coefficient [52]:

$$\begin{aligned} c_{4[PA]}^{Bj} &\approx -111 \quad ([1/2] \text{ PA}) \\ c_{4[PA]}^{Bj} &\approx -114 \quad ([2/1] \text{ PA}) \end{aligned} \quad (42)$$

and the error estimator in equation (41) with $a = 1$ yields

$$\delta_{[1/2]} \simeq -1/8; \quad \delta_{[2/1]} \simeq -1/4 \quad (43)$$

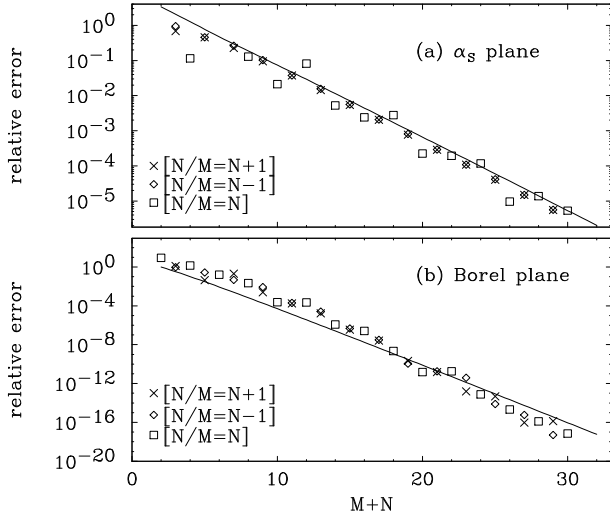


Figure 2. Relative errors in the $[N/M]$ Padé approximants (a) to the QCD vacuum polarization D-function, evaluated to all orders in the large- N_f approximation [56] (the rate of convergence agrees with expectations for a series with a discrete set of Borel poles), and (b) to the Borel transform of the D-function series, where the convergence is particularly striking. The straight lines correspond to the error formulae, eqs. (41) and (51), respectively.

These results can be combined to obtain

$$c_{4[PA]}^{Bj} = \frac{1}{2} \left(\frac{-111}{1 + \delta_{[1/2]}} + \frac{-114}{1 + \delta_{[2/1]}} \right) \approx -139 \quad (44)$$

which is very close to the ECH estimate [48]

$$c_{4[ECH]}^{Bj} \simeq -130 \quad (45)$$

A second application of PA's is to “sum” the full perturbative series. The latter is ambiguous if the perturbative series possesses an infrared renormalon singularity, i.e. a divergence of the form in equation (36) with $K > 0$. Consider the following toy example:

$$\sum_0^\infty n! x^n = \int_0^\infty \frac{e^{-t}}{1-xt} dt = \frac{1}{x} \int_0^\infty \frac{e^{-y/x}}{1-y} dy \quad (46)$$

which exhibits an infrared renormalon pole at $y = 1$. One possible way to define the ambiguous integral on the right hand side of equation (46) is via the Cauchy principle value prescription [58]. We see in Fig. 3 that the errors in the Padé “Sums” (PS's) $[N/M](x)$ are smaller than the truncated perturbative series

$$\sum_{n=0}^{N+M} n! x^n \quad (47)$$

when $n < n_{opt}$, which is 5 in this example. You will notice in Fig. 3 that the errors in the PS's become

unstable for large n : this is because of nearby poles in the denominator of equation (38) which are not important for small n . Also shown in Fig. 3 as “combined method” is a systematic approach to treating these poles and optimizing the PS's for large n , which is described elsewhere [59].

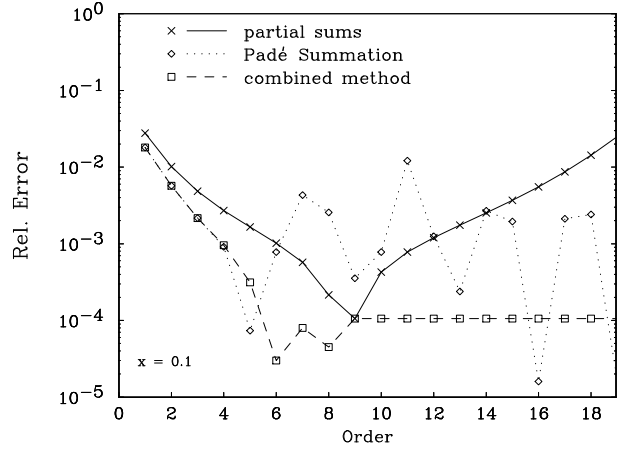


Figure 3. The relative errors between partial sums of the series $S(x) = \Sigma n! x^n$ and the Cauchy principal value of the series (solid line) is compared with the relative errors of Padé Sums (dotted line). We see that the relative errors of the Padé Sums are smaller than those of the partial sums in low orders, fluctuate in an intermediate régime, and are again more accurate than the partial sums in higher orders. The fluctuations are associated with nearby poles in the Padé Sums, that may be treated by the “combined method” mentioned in the text, shown as the dashed line.

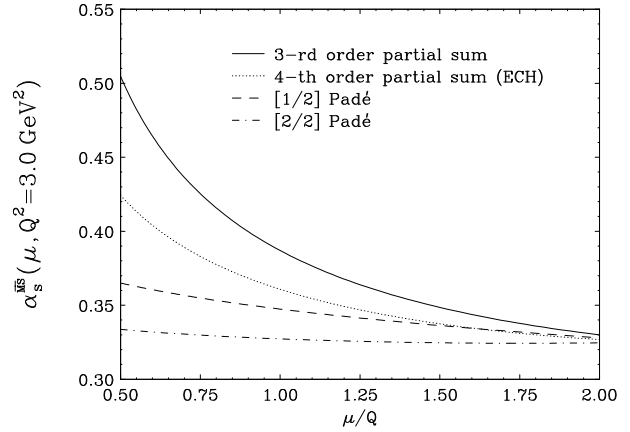


Figure 4. The scale dependence of $\alpha_s(3\text{GeV}^2)$ obtained from a fixed value $f(x) = (6/g_A) \times 0.164 = 0.783$, (cf. eq. (54)), for $Q/2 < \mu < 2Q$, using the naïve third- and fourth-order perturbative series and the $[1/2]$ and $[2/2]$ PS's.

Evidence that the PS's for the Bjorken sum rule provide a good estimate of the perturbative correction

factor in equation (16) is provided by the study of the renormalization scale dependence. We see in Fig. 4 that the renormalization scale dependence of the $[2/2]$ PS is much smaller than that of the $[2/1]$ and $[1/2]$ PS's, which is in turn much smaller than that of the naïve perturbation series evaluated to third order. We recall [60] that the full correction factor should be scale-independent, and interpret Fig. 4 as indicating that the PS's may be very close to the true result.

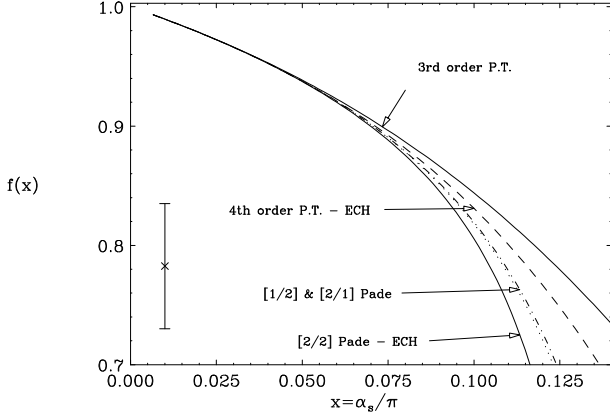


Figure 5. Different approximations to the Bjorken sum rule correction factor $f(x)$, third-order and fourth-order perturbation theory, $[1/2]$, $[2/1]$ and $[2/2]$ Padé Sums are compared. Also shown as a vertical error bar is the value of $f(x)$ we extract from the available polarized structure data (54).

Fig. 5 shows the estimates of the perturbative QCD correction to the Bjorken sum rule obtained in various different approximations, including third-order perturbation theory, fourth-order perturbation theory estimated using the ECH technique and the $[2/1]$, $[1/2]$ and $[2/2]$ PS's. We interpret the latter as the best estimator, and take the difference between it and the $[2/1]$ and $[1/2]$ PS's as a theoretical uncertainty. Also shown in Fig. 5 is the experimental error on this quantity, as extracted from the combined analysis of the available experimental data discussed in the next section.

More information can be extracted by considering PA's in the Borel plane. The Borel transform of a perturbative series is defined by

$$S(x) = \sum_{n=0}^{\infty} c_n x^n \xrightarrow{\text{Borel}} \tilde{S}(y) \equiv \sum_{n=0}^{\infty} \tilde{c}_n y^n : \tilde{c}_n = \frac{c_{n+1}}{n!} \left(\frac{4}{\beta_0} \right)^{n+1} \quad (48)$$

where $\beta_0 = (33 - 2N_f)/3$. A discrete set of renormalon singularities would show up as a set of finite-order poles

in this plane

$$\frac{r_k}{(y - y_k)^P} \quad (49)$$

The PA's in equation (38) are clearly well suited to find the locations y_k and the residues r_k of such poles. In the case of a perturbative series dominated by a finite set of L renormalon singularities, a sufficiently high-order PA will be *exact*

$$[M/N](y) = \tilde{S}(y) : \quad \text{for } M + N > L_0 \quad (50)$$

for some $L_0 \propto L$. Generically, in any case where the quantity analogous to eq. (39), $\tilde{\epsilon}_n \simeq 1/n^2$, the error analogous to (40) is given by

$$\tilde{\delta}_{[M/M]} \simeq - \frac{(M!)^2}{L^{2M}} \quad (51)$$

This prediction of very rapid convergence is confirmed in Fig. 2b [51] in the case of the QCD vacuum polarization D function evaluated in the large N_f limit [56], which has an infinite number of renormalon poles.

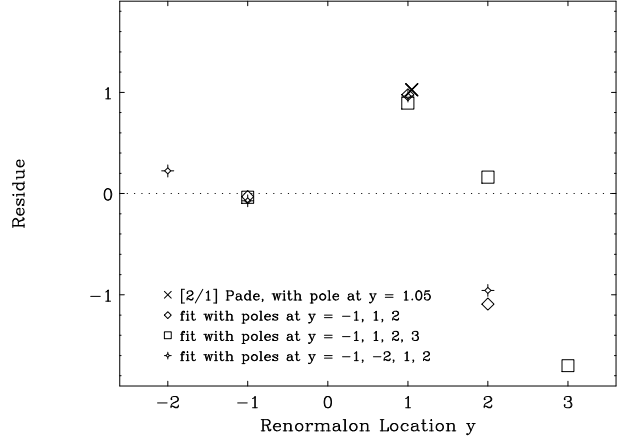


Figure 6. The locations and residues of poles in the $[2/1]$ PA and in rational-function fits to the Borel transform of the first four terms in the perturbation series for the Bjorken sum rule. We note that the location of the lowest-lying infrared renormalon pole is estimated accurately by Padé Approximants in the Borel plane, and that its residue is stable in the different fits.

The power of the application of PA's in the Borel plane is shown in Fig. 6, where we see that the $[2/1]$ PA of the Borel-transformed Bjorken series has a pole at $y = 1.05$. The agreement with the expectation of a first infrared renormalon pole at $y = 1$ is striking. Fig. 6 also shows other rational-function approximations to the Borel-transformed Bjorken series. We see evidence that this is dominated by a strong infrared renormalon pole at $y = 1$, that there may be a weak ultraviolet renormalon pole at $y = -1$ and possibly another pole at $y = 2$.

The ambiguity in the definition of the perturbative Bjorken series associated with the $y = 1$ renormalon singularity corresponds to a possible $1/Q^2$ correction of magnitude [52]

$$\Delta (\Gamma_1^p - \Gamma_1^n) = \pm \frac{|g_A|}{6} 0.98 \pi \frac{\Lambda^2}{Q^2} \quad (52)$$

It is expected that the QCD correction to the Bjorken sum rule should include a higher-twist correction of similar form with magnitude [61, 62, 63]

$$\Delta_{HT} (\Gamma_1^p - \Gamma_1^n) = - \frac{0.02 \pm 0.01}{Q^2} \quad (53)$$

The perturbative ambiguity in equation (52) is cancelled by a corresponding ambiguity in the definition of the higher-twist contribution. In the next section we will treat equation (53) as a correction (with error) to be applied to the perturbative QCD factor shown in Fig. 5.

We have also compared PA's to the predictions of commensurate scale relations within the framework of ref. [50]. The predictions of the two approaches are numerically very similar, and we give formal reasons in ref. [59] why we believe that this should be so. However, we shall not use commensurate scale relations in the data analysis of the next section.

1.5. Numerical analysis of the Bjorken sum rule

Table I shows the data on the integrals $\Gamma_1^{p,n}$ currently available from experiments at CERN and SLAC [17],[64]-[66]. We do not attempt to correct these numbers for any of the effects discussed in section 1.4, such as the Q^2 -dependence of the asymmetry A_1 , the extrapolation to low x , or the transverse polarization asymmetry. We do not believe that any of these effects will change any of the data outside their quoted errors. We choose to evaluate the Bjorken sum rule at $Q^2 = 3 \text{ GeV}^2$, which requires rescaling all the data as described in ref. [52].

Table I

$\Gamma_1^{p,n,d}$ currently available from experiments at CERN and SLAC.

experiment	target	Γ_1
E142	n	-0.045 ± 0.009
E143	p	0.124 ± 0.011
E143	d	0.041 ± 0.005
SMC	d	0.023 ± 0.025
SMC ('94)	d	0.030 ± 0.011
SMC	p	0.122 ± 0.016
EMC	p	0.112 ± 0.018

All experimental data have been evolved to $Q^2 = 3 \text{ GeV}^2$.

The following is the combined result that we find for the Bjorken sum rule:

$$\Gamma_1^p(3\text{GeV}^2) - \Gamma_1^n(3\text{GeV}^2) = 0.164 \pm 0.011 \quad (54)$$

which is indicated by a vertical error bar in the lower left corner of Fig. 5. Comparing this value with the [2/2] PS estimate also shown there, we find

$$\alpha_s(3 \text{ GeV}^2) = 0.328_{-0.037}^{+0.026} \quad (55)$$

which becomes

$$\alpha_s(M_Z^2) = 0.119_{-0.005}^{+0.003} \pm \dots, \quad (56)$$

when we run α_s up to M_Z^2 using the three-loop renormalization group equation. The errors quoted in equations (55) and (56) are purely experimental, and the second \pm sign in equation (56) indicates that further theoretical systematic errors must be estimated. Those we have evaluated include that associated with the renormalization scale dependence shown in Fig. 4 (± 0.002), the difference between the [2/2] and [2/1], [1/2] PS's (± 0.002), and the correction due to the higher-twist estimate in equation (53) (-0.003 ± 0.002), whereas the error in the running of α_s is found to be negligible. Combining these estimates with equation (56), we find [52]

$$\alpha_s(M_Z^2) = 0.116_{-0.005}^{+0.003} \pm 0.003, \quad (57)$$

The stability of this result is indicated in Fig. 7, where we exhibit the values of $\alpha_s(M_Z^2)$ obtained using different orders of perturbation theory, compared with our result (57) obtained using the [2/2] PS. Also indicated is the shift induced by the higher-twist correction, which lies within our error bars.

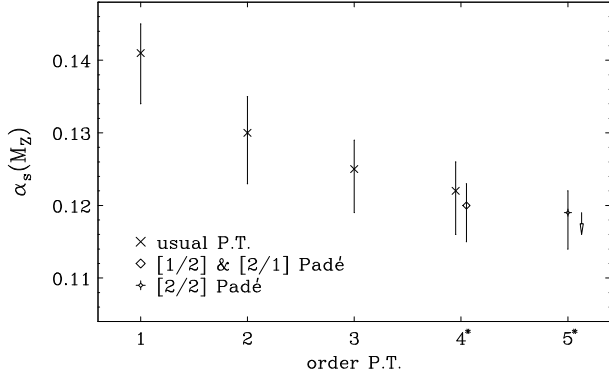


Figure 7. Values of $\alpha_s(M_Z^2)$ obtained using different orders of perturbation theory, compared with our result (56), obtained using the [2/2] PS. The size of the shift induced by higher-twist correction is (53) indicated by a downward arrow to the right of the [2/2] point.

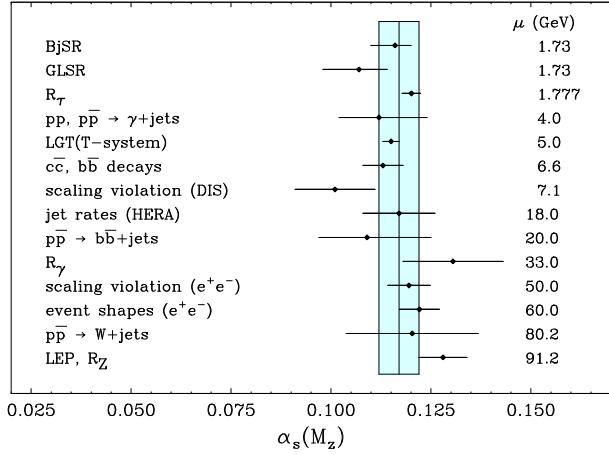


Figure 8. Compilation of world data on α_s from different sources (adapted from Ref. [68]).

As can be seen from the compilation in Fig. 8, our central value for $\alpha_s(M_Z^2)$ is quite compatible with other determinations and the world average, which is quoted to be [1, 67, 68]

$$\alpha_s(M_Z^2) = 0.117 \pm 0.005 \quad (58)$$

Indeed, the error quoted in equation (57) is quite competitive with the most precise determinations of $\alpha_s(M_Z^2)$ that are available. Moreover, we note that plenty of precise new data will soon be available from the SMC experiment at CERN, the E154 and E155 experiments at SLAC, and the HERMES experiment [69] at DESY. In the longer run, experiments with a polarized proton beam at HERA will provide valuable information on the behaviour of g_1 at low x , as well as on its Q^2 -dependence at fixed x .

1.6. Decomposition of the Nucleon Spin

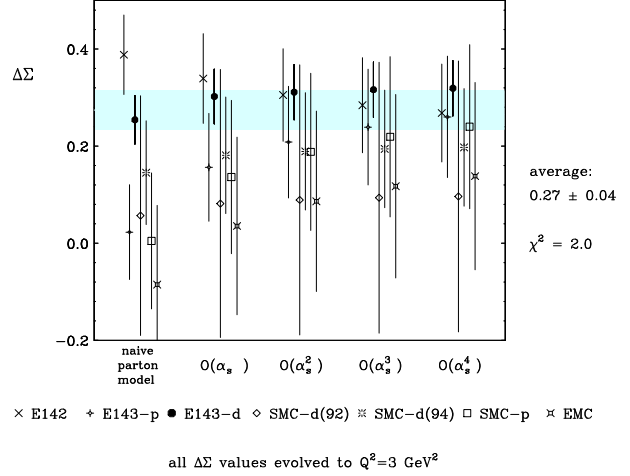


Figure 9. The values of $\Delta\Sigma(Q^2=3 \text{ GeV}^2)$ extracted from each experiment, plotted as functions of the increasing order of QCD perturbation theory used in obtaining $\Delta\Sigma$ from the data (from Ref. [62], updated with most recent data).

So far, we have only discussed the combination $\Gamma_1^p - \Gamma_1^n$ which enters in the Bjorken sum rule. The individual $\Gamma_1^{p,n}$ can be used as in equation (13), though not forgetting the perturbative QCD corrections in equation (17), to extract the individual Δq . As is seen in Fig. 9, the different experiments on both proton and neutron targets are all highly consistent, *once the perturbative QCD corrections are taken into account*. Some time ago, it appeared as if the neutron data from E142 might be at variance with the other data points. However, this is no longer the case if all the higher-order corrections in equation (17) are taken into account, and the latest evaluations [66] of the E142 data indicate a different preliminary value of Γ_1^n , as seen in Table I. Making a global fit, we find

$$\begin{aligned} \Delta u &= 0.82 \pm 0.03 \pm \dots \\ \Delta d &= -0.44 \pm 0.03 \pm \dots \\ \Delta s &= -0.11 \pm 0.03 \pm \dots \end{aligned} \quad (59)$$

and

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.27 \pm 0.04 \pm \dots \quad (60)$$

where the second \pm sign indicates that further theoretical and systematic errors remain to be assigned. These include higher-twist effects, errors in the extrapolation to low x which is more complicated than for the nonsinglet combination of structure functions appearing in the Bjorken integrand, the possible Q^2 -dependence of A_1 , etc.. We believe that these errors

may combine to be comparable with the errors quoted in equations (59),(60), but prefer not to quote definitive ranges for the Δq until all these errors are controlled as well as those appearing in the Bjorken sum rule.

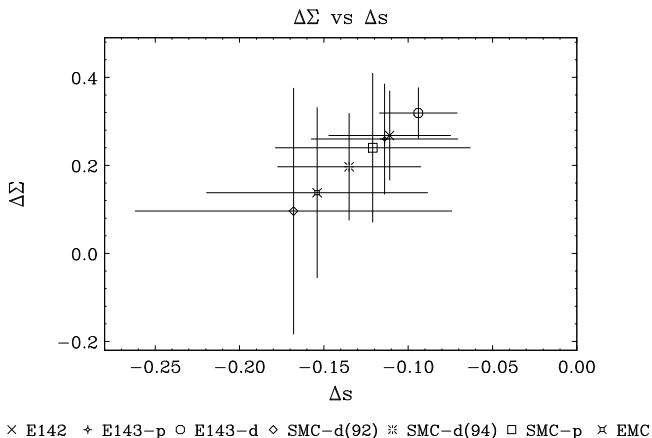


Figure 10. The values of $\Delta\Sigma$ and Δs extracted from each experiment, plotted against each other. All data have been evolved to common $Q^2 = 3 \text{ GeV}^2$. The clear linear correlation between $\Delta\Sigma$ and Δs results from the linear relations (9),(10),(17).

One may also get a feeling for the expected range of $\Delta\Sigma$ and Δs by plotting the results for these two observables extracted from each of the existing experiments, as shown in Fig. 10.

2. Outlook

In this talk we have concentrated on the phenomenological analysis of the data on polarized structure functions presently available. As we have seen, these tell a remarkably consistent story, once higher-order QCD corrections are included. We have not addressed in great detail here the theoretical interpretation of the data, nor their spin-offs in hadron physics and elsewhere, nor possible future developments in this field. In fact, these measurements provide valuable insights into important issues in non-perturbative QCD, such as the rôle of chiral symmetry in nucleon structure [19], the axial anomaly and the $U(1)$ problem [20]-[22], and the relationship between current and constituent quarks [70]-[75] which are provoking lively theoretical debates (see Ref. [76] for a recent application to the pion structure).

The polarized structure function data support previous indications from the π -nucleon σ -term and elsewhere that strange quarks in the nucleon wave function cannot be neglected, with interesting implications for the analysis of recent data from LEAR on ϕ production in proton-antiproton annihilation [77]. Among other spin-offs, we recall that the axial-current matrix elements extracted

from polarization data determine scattering matrix elements for candidate dark matter particles such as the lightest supersymmetric particle [78] and the axion [62].

In the future, we look forward to the completion of the SMC programme and its possible HMC successor at CERN, the E154 and E155 experiments at SLAC, data from the HERMES experiment at HERA, the polarized proton programme at RHIC, and possible polarized electrons and protons in the HERA ring. The tasks of these experiments will include the determination of the gluon contribution to the nucleon spin, the elucidation of the Q^2 dependence of A_1 , and the low- x behaviour of g_1 . These will continue to fuel activity in this interesting field for the foreseeable future, which will lead us to a deeper understanding of the nucleon, an object we thought we knew so well, but which reveals a new face when it spins.

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